1. Set the hair line over the base 'b' on appropriate LL scale.
2. Place the left or right index of the C scale under the hair line.
3. Reset the hair line over the number 'N' on its appropriate LL scale.
4. Under the hair line read off the value for x on the C scale and locate the decimal point according to the LL scales used.

Note:

(a) To solve the *equation* $b^{\frac{1}{x}} = N$ for x, we follow the same method as outlined above, except in step 4 the value of x is read off the CI scale.

(b) To solve the *equation* $b^{kx} = N$ for x, the following method can be used—

Example: $e^{1.4x} = 9$

1. Set the hair line over e on the LL_3 scale.
2. Place the 1.4 of the CIF scale under the hair line. (We can also use CI scale here if suitable).
3. Reset the hair line over 9 on the LL_3 scale.
4. Under the hair line read off 1.57 on the CF scale as the value for x.
   (When the CI scale is used in 2, the answer in 4 will be on the C scale).

EXERCISE 19(e)

(i) $\log_5 1.9$ =
(ii) $\log_8 24$ =
(iii) $\log_2 57$ =
(iv) $\log_3 0.52$ =

(v) $\log_{1.5} 2$ =
(vi) $\log_{15} 1.3$ =
(vii) $\log_5 260$ =
(viii) $\log_{2.5} 17$ =

Find x in the following:

(ix) 5^x = 30
(x) 12^x = 76
(xi) 15^x = 3.5
(xii) 456^x = 21
(xiii) 0.3^x = 0.95

(xiv) 0.16^x = 0.045
(xv) $2.5^{\frac{1}{x}}$ = 1.02
(xvi) $16e^x$ = 48
(xvii) e^{-x} = 64
(xviii) e^{-3x} = 0.45

20

ROOT (W) SCALES

20.1   THE FORM OF THE W SCALES

The Faber-Castell Slide Rules 2/83N and 62/83, incorporate these scales. In the case of 2/83N, the W scales are actually 50 cm. "C and D scales" cut in half at $\sqrt{10}$ (i.e. 3.162) and fitted on a 25 cm (10 inch) Slide Rule. For the 62/83 they are 25 cm. "C and D scales" cut in half and fitted on a 12.5 cm (5 inch) Slide Rule. For the 2/83N, the W scales give approximately four significant figures of accuracy, while the 62/83 can be read to about three figures, thus making calculations with the W scales on this 12.5 cm (5 inch) Slide Rule as accurate as the normal C and D scales on a 25 cm (10 inch) Slide Rule.

20.2   MULTIPLICATION

The procedure differs somewhat from that used with the C and D scales. Depending on the numbers involved we have to use either the left (1) or right (10) black index marks, or one of the red index marks located on the right hand end of the W'_1 and W_1 scales, or the left hand end of the W_2 and W'_2 scales. The following examples show each of the four possible variations.
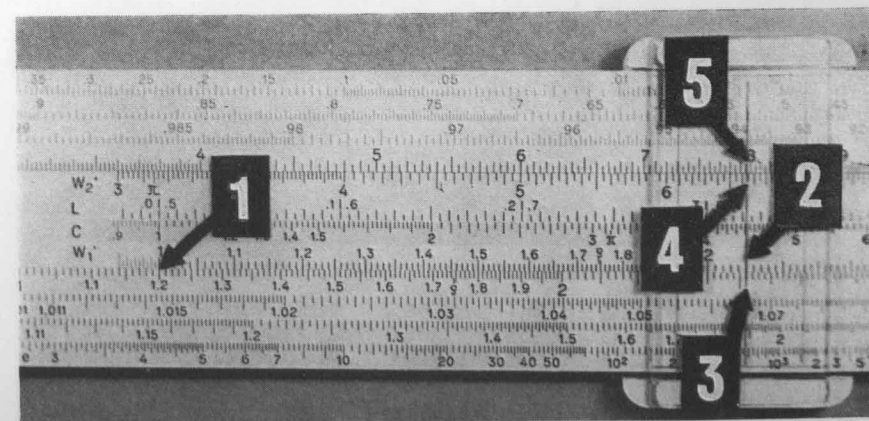

Figure 20.1

Example 1: $1.2 \times 21 = 25.2$ (Fig. 20-1)

1. Place the left (black) index of the W'_1 scale over 1.2 on the W_1 scale.
2. Set the hair line over 21 on the W'_1 scale.
3. Under the hair line read off 25.2 on the W_1 scale as the answer.

   (Note, under the hair line 6.64 (at 4) on the W'_2 scale gives $1.2 \times 6.64 = 7.97$ (at 5) on the W_2 scale.)
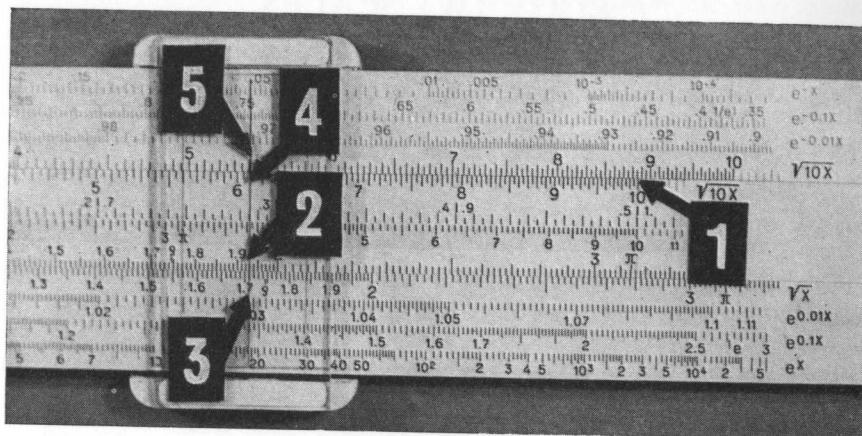

Figure 20.2

Example 2: $8.86 \times 19.3 = 171$ (Fig. 20-2)

1. Place the right (black) index of the W'_2 scale over 8.86 on the W_2 scale.
2. Set the hair line over 19.3 on the W'_1 scale.
3. Under the hair line read off 171 on the W_1 scale as the answer.

   (Note, under the hair line 6.1 (at 4) on the W'_2 scale gives $8.86 \times 6.1 = 54.1$ (at 5) on the W_2 scale.)
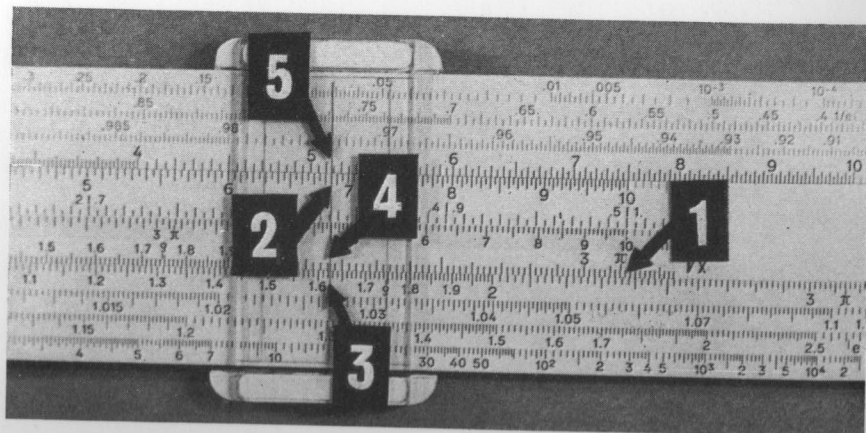

Figure 20.3

Example 3: $2.37 \times 6.85 = 16.25$ (Fig. 20-3)

1. Place the right (red) index of the W'_1 scale over 2.37 on the W_1 scale.
2. Set the hair line over 6.85 on the W'_2 scale.
3. Under the hair line read off 16.25 on the W_1 scale as the answer.

   (Note, under the hair line 21.65 (at 4) on the W'_1 scale gives $2.37 \times 21.65 = 51.4$ (at 5) on the W_2 scale.)
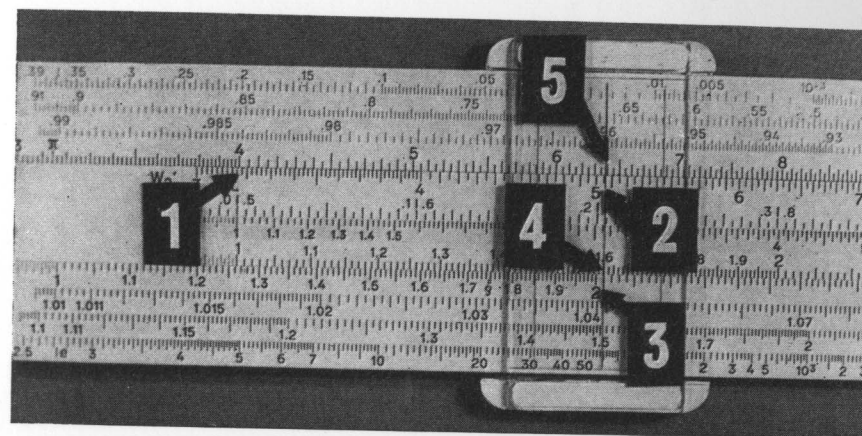

Figure 20.4

Example 4: $39.9 \times 50.6 = 2,020$ (Fig. 20-4)

1. Place the left (red index of the W'_2 scale over 39.9 on the W_2 scale.
2. Set the hair line over 50.6 on the W'_2 scale.
3. Under the hair line read off 2,020 on the W_1 scale as the answer.

   (Note, under the hair line 1.6 (at 4)) on the W'_1 scale gives $39.9 \times 1.6 = 63.9$ (at 5) on the W_2 scale.)

The following table summarises these procedures for easy reference.

Set H.L. over-	Under H.L. place-	Reset H.L. over-	Under H.L. answer-
a on W_1 scale	black index of W'_1	b on W'_1 scale	ab on W_1 scale
		b W'_2	ab W_2
	red index of W'_1	b W'_1	ab W_2
		b W'_2	ab W_1
a on W_2 scale	black index W'_2	b W'_1	ab W_1
		b W'_2	ab W_2
	red index W'_2	b W'_1	ab W_2

RULES FOR MULTIPLICATION

(i) When we use the *black index marks* (i.e. 1 or 10) of the W'_1 or W'_2 scales, the answer is read off the W_1 or W_2 scales respectively. (i.e. the *scale* on the body of the Slide Rule *adjacent* to the second factor of the multiplication.)

(ii) When we use the *red index marks* of the W'_1 or W'_2 scales, the answer is read off the W_2 or W_1 scales respectively. (i.e. the *scale* on the body of the Slide Rule *opposite* to the second factor of the multiplication.)

EXERCISE 20(a)

(i) 0.11×175	=	(vi) 650×6.5 =
(ii) 50.2×31.8	=	(vii) 0.945×61.5 =
(iii) 0.14×0.26	=	(viii) 111×0.941 =
(iv) 25.6×142	=	(ix) $2 \times \pi$ =
(v) 18.3×0.031	=	(x) 40.25×51.5 =

(For continued multiplication hold the progressive answer with the hair line as usual.)

(xi) $17 \times 28 \times 46$	=	(xiv) $36.2 \times 38 \times 0.042$ =
(xii) $0.18 \times 104 \times 0.043$	=	(xv) $80.4 \times 0.171 \times 0.0316$ =
(xiii) $62 \times 1.2 \times 0.47$	=	(xvi) $29.2 \times 4.75 \times 132 \times 72$ =

20.3 DIVISION

Division is, as usual, the reverse procedure to multiplication. The following examples show two of the possible cases.
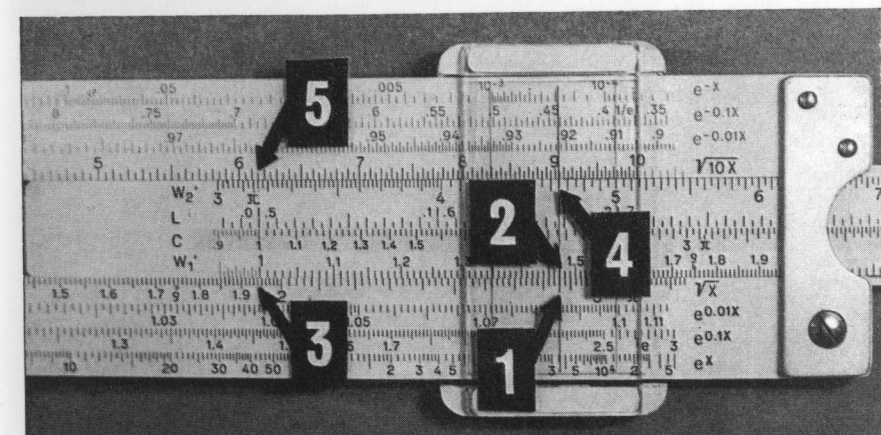

Figure 20.5

Example 1: $28.6 \div 147 = 0.1945$ (Fig. 20-5)
1. Set the hair line over 28.6 on the W_1 scale.
2. Place the 147 of the W'_1 scale under the hair line.

3. Below the left (black) index of the W'_1 scale read off the answer as 0.1945 on the W_1 scale.

(Note, under the hair line 4.65 (at 4) on the W'_2 scale gives above the left (red) index of W'_2 scale, $28.6 \div 4.65 = 6.15$ (at 5) on the W_2 scale.)
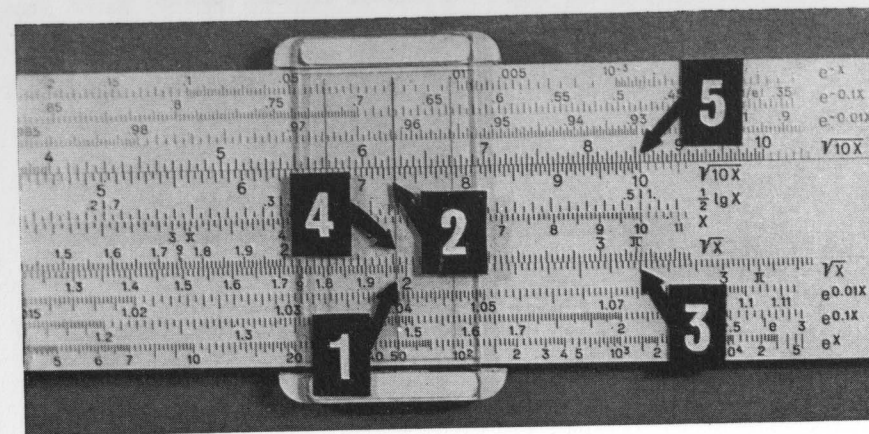

Figure 20.6

Example 2: $19.75 \div 73 = 0.2705$ (Fig. 20-6)
1. Set the hair line over 19.75 on the W_1 scale.
2. Place the 73 of the W'_2 scale under the hair line.
3. Below the right (red) index of the W'_1 scale read off the answer as 0.2705 on the W_1 scale.
(Note, under the hair line 2.31 (at 4) on the W'_1 scale gives, above the right (black) index of the W'_2 scale, $19.75 \div 2.31 = 8.55$ (at 5) on the W_2 scale.)

All the possibilities for division are summarised in the following table.

Set H.L. over–	Under H.L. place	Reset H.L. over—	Under H.L.	answer—
a on W_1 scale	b on W'_1 scale	black index of W'_1	$a \div b$ on W_1 scale	
		black index	W'_2 $a \div b$	W_2
	b W'_2	red index	W'_1 $a \div b$	W_1
		red index	W'_2 $a \div b$	W_2
a on W_2 scale	b W'_2	black index	W'_2 $a \div b$	W_2
		black index	W'_1 $a \div b$	W_1
	b W'_1	red index	W'_2 $a \div b$	W_2
		red index	W'_1 $a \div b$	W_1

RULES FOR DIVISION

(i) When the numbers involved in the division are located on *adjacent scales,* the answer is read off the W_1 or W_2 scales under either of the *black index marks.*

(ii) When the numbers involved in the division are located on scales on *opposite sides of the slide,* the answer is read off the W_1 or W_2 scales under either of the *red index marks.*

EXERCISE 20(b)

(i) $360 \div 18$ $=$

(ii) $4,800 \div 0.6$ $=$

(iii) $12.25 \div 35$ $=$

(iv) $43.75 \div 0.0304$ $=$

(v) $3,025 \div 55$ $=$

(vi) $1,925 \div 17.5$ $=$

(vii) $\pi \div 6$ $=$

(viii) $93 \div 9,600$ $=$

(ix) $\dfrac{219}{17 \times 28}$ $=$

(x) $\dfrac{35}{0.12 \times 0.47}$ $=$

(xi) $\dfrac{805}{(104 \times 0.043)}$ $=$

(xii) $1,406 \div 52^2$ $=$

20.4 SQUARES AND SQUARE ROOTS

A. SQUARES

For numbers on the W'_1 or W'_2 scales, their *squares* are found on the C scale. We use the W'_1 and W'_2 scales in preference to the W_1 and W_2 scales, as the former are on the slide with the C scale.
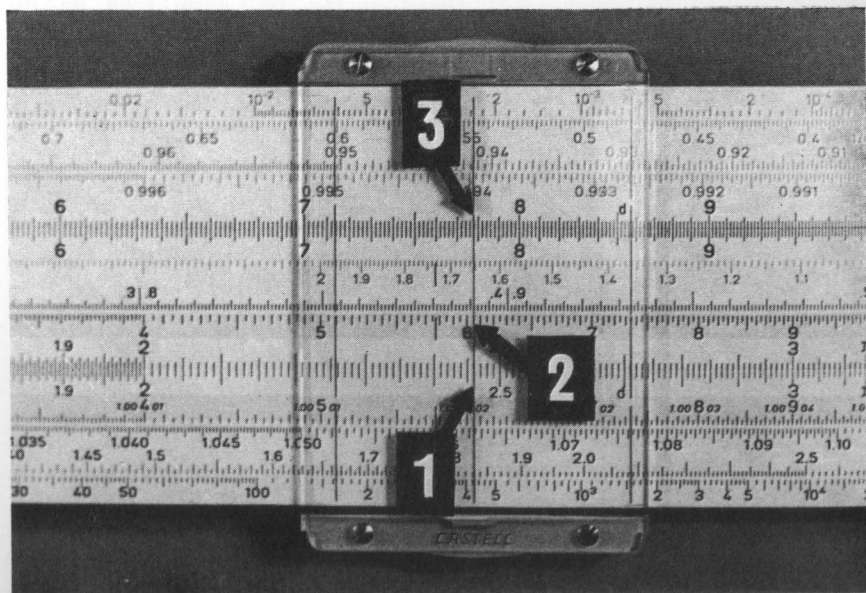

Figure 20.7

Example: $2.46^2 = 6.05$ (Fig. 20-7)

1. Set the hair line over 2.46 on the W'_1 scale.
2. Under the hair line read off 6.05 on the C scale as the answer.

Note: In Fig. 20-7, under the hair line the 7.78 (at 3) on the W'_2 scale would be read off the C scale (at 1) as $7.78^2 = 60.5$.

EXERCISE 20(c)

(i) 7.65^2 $=$ (v) 0.084^2 $=$

(ii) 0.9^2 $=$ (vi) 0.00022^2 $=$

(iii) 65^2 $=$ (vii) 30.25^2 $=$

(iv) 207^2 $=$ (viii) $(5.4 \times 10^3)^2$ $=$

B. SQUARE ROOTS

For a number on the C scale, its *square root* is read off the W'_1 scale if the number is between 1 and 10, or off the W'_2 scale if the number is between 10 and 100. For numbers larger than 100 or less than 1, we use the procedure as outlined in Unit 5.
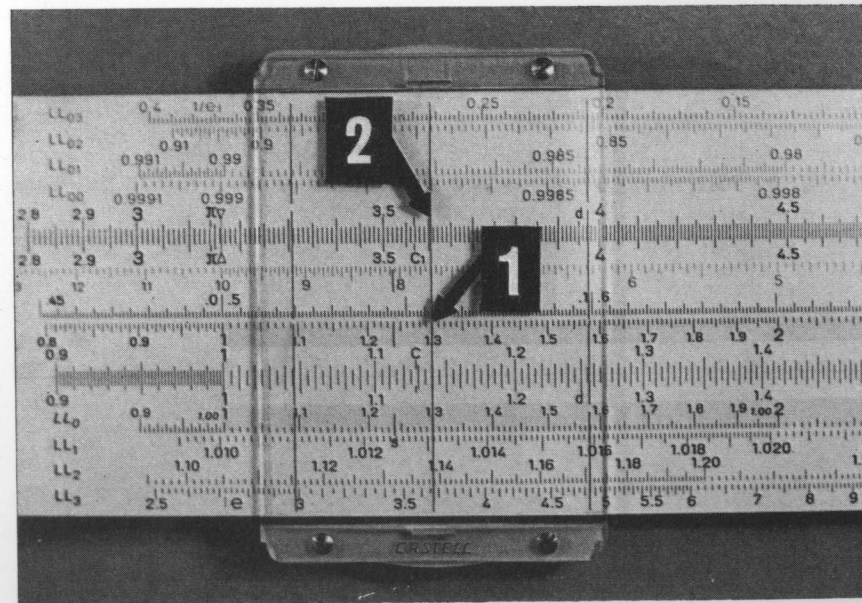

Figure 20.8

Example 1: $\sqrt{13} = 3.605$ (Fig. 20-8)

1. Set the hair line over 13 on the C scale. (As the number is between 10 and 100 we find its square root on the W'_2 scale.)
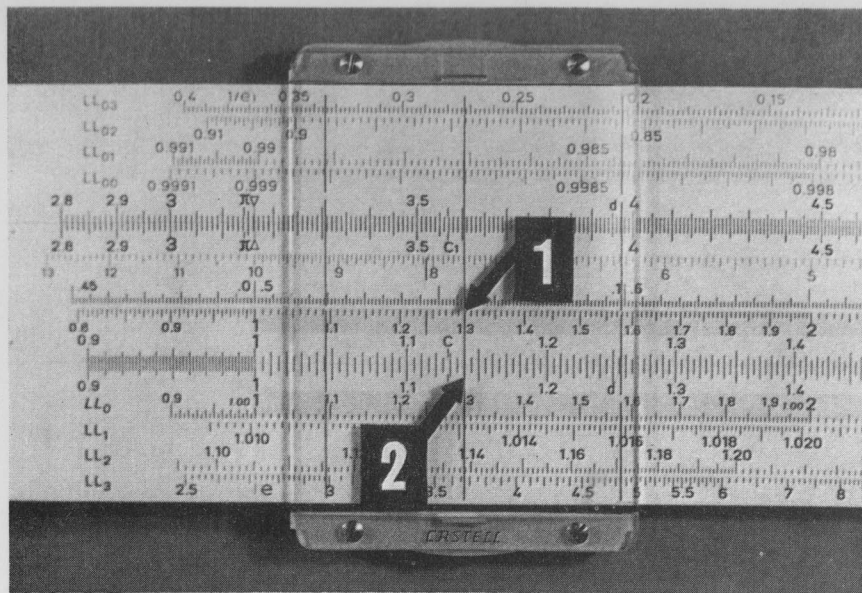2. Under the hair line read off 3.605 on the W'_2 scale as the answer.

Figure 20.9

Example 2: $\sqrt{130} = 11.4$ (Fig. 20-9)

1. Set the hair line over 130 on the C scale. (Express $\sqrt{130} = \sqrt{1.3 \times 100} = \sqrt{1.3} \times 10$, thus we find $\sqrt{1.3}$.)
2. Under the hair line read off 1.14 on the W'_1 scale as the value for $\sqrt{1.3}$.

$$\therefore \text{ answer} = 1.14 \times 10$$
$$= 11.4$$

Example 3: $\sqrt{0.098} = 313$

1. Set the hair line over 9 on the C scale. (Express $\sqrt{0.098} = \sqrt{\dfrac{98}{100}} = \sqrt{\dfrac{9.8}{10}}$, thus we find $\sqrt{9.8}$.)
2. Under the hair line read off 3.13 on the W'_1 scale as the value for $\sqrt{9.8}$.

$$\therefore \text{ answer} = \frac{3.13}{10}$$
$$= 0.313$$

EXERCISE 20(d)

(i) $\sqrt{6.25} =$		(vi) $\sqrt{2,450} =$	
(ii) $\sqrt{93.5} =$		(vii) $\sqrt{0.06} =$	
(iii) $\sqrt{1.125} =$		(viii) $\sqrt{0.143} =$	
(iv) $\sqrt{324} =$		(ix) $\sqrt{0.4} =$	
(v) $\sqrt{960} =$		(x) $\sqrt{0.0025} =$	

20.5 MISCELLANEOUS CALCULATIONS

The following tables list a number of calculations which make use of the W scales, and form a supplement to the table given in 10.3. For squares and square roots the appropriate W scale must be used according to the numbers involved.

Example	Set H.L. over-		Under H.L. place-		Reset H.L. over-		Under H.L. answer-
a^2b	a on	W scale	index of	W' scale	b on	C scale	on D scale
ab^2	a	D	index	C	b	W'	D
ab^4	b	W	a	CI	b	W'	D
a^2b^2	a	W	index	W'	a	W'	D
a^2b^2c	a	W	c	CI	b	W'	D
$\dfrac{a}{b^2}$	a	D	b	W'	index	W'	D
$\dfrac{a^2}{b^3}$	a	W	b	W'	b	CI	D
$\dfrac{a^2}{b}$	a	W	b	C	index	C	D
$\dfrac{a^3}{b}$	a	W	b	C	a	C	D
$\dfrac{1}{ab^2}$	a	DI	b	W'	index	W'	D
$\dfrac{1}{a^2b^2}$	a	W	index	W'	b	W'	DI
$\dfrac{1}{a^4}$	index	W	a	W'	index	W'	A
$\dfrac{a^2b}{c}$	a	W	c	C	b	C	D
$\dfrac{a^2b}{c^2}$	a	W	c	W'	b	C	D
$\dfrac{ab}{c^2}$	a	D	c	W'	b	C	D
$a\sqrt{b}$	a	W	index	W'	b	C	W
$\sqrt{ab}$	a	D	index	C	b	C	W
$\sqrt{abc}$	a	D	b	CI	b	C	W
$a^2\sqrt{b}$	a	W	index	W'	b	B	D
$\sqrt{\dfrac{a}{b}}$	a	D	b	C	index	C	W
$\dfrac{\sqrt{a}}{b}$	a	D	b	W'	index	W'	W
$\dfrac{a}{\sqrt{b}}$	a	W	b	C	index	W'	W
$\sqrt{\dfrac{ab}{c}}$	a	D	c	C	b	C	W

Example	Set H.L. over-	Under H.L. place-	Reset H.L. over-	Under H.L. answer-
$\sqrt{\dfrac{a}{bc}}$	a on D scale	b of C scale	c on CI scale	on W scale
$\dfrac{1}{\sqrt{ab}}$	a DI	index CI	b CI	W
$\dfrac{1}{a\sqrt{b}}$	index W	a W'	b CI	W
$a\sqrt{bc}$	b D	c CI	a W'	W
$a\sqrt{\dfrac{b}{c}}$	b D	c C	a W'	W
$\pi r^2 h$	r W	h CI	index C	DF
$\tfrac{1}{2}r^2\theta$	θ (degrees) ST 2	C	r W'	D
	θ (radians) D 2	C	r W'	D

Example	Set H.L. over-	Under H.L. answer-
a^4	a on W scale	on A scale
$\sqrt[4]{a}$	a A	W
a^6	a W	K
$\sqrt[6]{a}$	a K	W
$\dfrac{1}{\sqrt{a}}$	a DI	W
$\dfrac{1}{a^2}$	a W	DI
$\sqrt[4]{1-a^2}$	a P	W
$\sqrt{1-a^4}$	a W	P
$\sqrt{\sin\theta}$	θ S	W
$\sqrt{\tan\theta}$	θ T	W
$\sqrt{\dfrac{a}{\pi}}$	a DF	W
πr^2	r W	DF

EXERCISE 20(e)

(i) 6.3^4 =

(ii) $\sqrt[4]{21.6}$ =

(iii) 2.15^6 =

(iv) $\sqrt[6]{583}$ =

(v) $\sqrt{\sin 35°}$ =

(vi) $\dfrac{2.35^2}{3.91}$ =

(vii) $\dfrac{37.6}{\sqrt{95}}$ =

(viii) 45×2.7^4 =

(ix) $\sqrt{1.09 \times 2.66}$ =

(x) $0.85^2 \sqrt{64.9}$ =

(xi) $\dfrac{21.6 \times 39.1}{17.4^2}$ =

(xii) $\dfrac{1}{11.3^4}$ =

(xiii) $\dfrac{1}{9.3^2 \times 1.3^2}$ =

(xiv) $\sqrt{\dfrac{11.7}{19.4}}$ =

(xv) $21.2^2 \times 19.1^2 \times 0.007$ =

(xvi) $\dfrac{\sqrt{139}}{45.3}$ =

(xvii) $\dfrac{65^2}{24.2}$ =

(xviii) $\sqrt{\tan 69°}$ =

(xix) $\pi \times 6.3^2 \times 11.9$ =

(xx) $\tfrac{1}{2} \times 6.8^2 \times 1.53$ =